Inverse Functions

October 7, 2009

1 One-to-One Functions

A one-to-one function is a function that satisfies the INVERSE NO-AMBIGUITY CONDITION: for every output, there is exactly one input. If a function \( f \) is one-to-one, then it has an inverse that is denoted by \( f^{-1} \).

The following tables represent functions whose inputs are \( x \) and whose outputs are \( y \). For each table, answer the following questions: (i) is \( f \) one-to-one, and if not give an example of an output that comes from more than one input, and (ii) does the table determine \( x \) as a function of \( y \)? If so, that function is the inverse function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>143</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>12</td>
<td>23</td>
<td>-19</td>
<td>-69</td>
<td>143</td>
</tr>
</tbody>
</table>

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<th>143</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>97</td>
<td>0</td>
<td>97</td>
<td>1</td>
</tr>
</tbody>
</table>

2 Inverse Functions

For each of the following equations, solve for \( x \). Does the result define \( x \) as a function of \( y \)?

1. \( y = \sqrt{x} \)

2. \( y = 3x - 8 \)

3. \( y = 2x^2 - 5 \)
4. \( y = \sqrt[3]{2x - 7} \)

5. \( y = \frac{1}{x} \)

6. \( y = |x| \)

7. \( y = x + 2 \)

8. \( y = 3x \)

3 Inverse Function Notation

If \( f \) is a one-to-one function, its inverse function is denoted with \( f^{-1} \). This is **NOT** an exponent! It does **NOT** mean \( 1/f \). It means nothing more and nothing less than the inverse function for \( f \). It is used in the same way \( f \) is used.

1. The function \( f(x) = 5x - 9 \) is one-to-one. What is \( f^{-1}(8) \)?

2. If the function \( f \) is one-to-one, what is \( f(f^{-1}(8)) \)?

3. If \( f \) is a one-to-one function and \( f(2) = 5 \), what is \( f^{-1}(5) \)?

4. \( \star \) Suppose \( f \) is one-to-one and has its domain given by \([1, \infty)\) and its range given by \((-\infty, 5]\). What is the domain and what is the range of \( f^{-1} \)?