Universal Properties of Shapes
Bell ringer

• Write down as many 3-dimensional shapes as you can in 60 seconds.

• What did you get?
Shapes in biology

• What shape is a human?
Shapes in biology

• What shape is a dog?
Shapes in biology

• What shape is a cactus?
Do these shapes have any common properties?

Figure 1. A Cubical Critter

horse splashesu For the resistance presented to movement by the air is proportional to the surface of the moving objectu Divide an animal's lengths breadths and height each by ten; its weight is reduced to a thousandths but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

An insects therefores is not afraid of gravity; it can fall without dangers and can cling to the ceiling with remarkably little troubleu It can go in for elegant and fantastic forms of support like that of the daddytlonglegsu But there is a force which is as formidable to an insect as gravitation to a mammalu This is surface tensionu A man coming out of a bath carries with him a film of water of about onetfiftieth of an inch in thicknessu This weighs roughly a poundu A wet mouse has to carry about its own weight of wateru A wet fly has to lift many times its own weight ands as everyone knowss a fly once wetted by water or any other liquid is in a very serious position indeedu An insect going for a drink is in as great danger as a man leaning out over a precipice in search of foodu If it once falls into the grip of the surface tension of the water that is to says gets wet it is likely to remain so until it drownsu A few insectss such as watertbeetless contrive to be unwettable; the majority keep well away from their drink by means of a long probosisu “

To flush out this comments mathematicallys we can begin simply by viewing all organisms as “cubical critters” that are characterized solely be their size d i f f e r e n c e s o Fig ypu If L is the length of one side of the cubical critters then it has a surface area of 6 \( L^2 \) cm \(^2\) sand a volume of \( L^3 \) m \(^3\)u Furthermores if we assume that these cubical critters are “ugly bags of mostly waters” then a critter of length \( L \) weighs \( m = L^3 \) gramsu Hences surface area is proportional to \( m^2/3 \) and the ratio of mass to surface area is proportional to \( m^{-1/3} \). Hences the larger you are the harder you fallu Converselys the ratio of surface area to mass is \( m^{-1/3} \). Therefores the smaller you are the more water weight you carry per unit biomass when you get wetu

There are many data sets which can be modeled by scaling lawsu For instances one of my favorite data sets is the mass lifted by an olympic weightlifter versus his body mass oFig zpu Assuming Olympic weightlifters 2 Star Trek fans may remember this line as an alien’s description of humans that are mostly water encased in a bag of skin. The “ugly” part is a matter of extraterrestrial taste.

Schreiber (2013) “Motivating Calculus with Biology”
“On being the right size”

Gravity- To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes. An insect is not afraid of gravity; it can fall without danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-longlegs.

But there is a force which is as formidable to an insect as gravity to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a wet fly is in very serious danger. An insect going for a drink is in as great a danger as a man leaning out over a cliff in search of food. If it once falls into the grip of the surface tension of the water, that is to say, gets wet it is likely to remain so until it drowns.

Modified from Haldane (1926), “On being the right size”
Properties of Volume

• Do part 1 of your activity packet (calculating the volume of each shape).
Hypothesis

- For each shape, there is a dilation of 2 and a dilation of 3.
- Make a hypothesis: How will the volume change when there is a dilation of 2 or 3?
Properties of Volume

• When a sphere had a dilation of 2, how much did its volume increase?
• What about a dilation of 3?
• When a cone had a dilation of 2, how much did its volume increase?
• What about a dilation of 3?
• When a cylinder had a dilation of 2, how much did its volume increase?
• What about a dilation of 3?
Properties of Volume

- What are the common properties of increasing volume?

Figure 1.

A Cubical Critter
Why does this work?
What does this mean for biology?

• Taller animals weigh MUCH more.
Properties of Surface Area

• Do part 2 of your activity packet (calculating the surface area of each shape).
Hypothesis

• For each shape, there is a dilation of 2 and a dilation of 3.
• Make a hypothesis: How will the surface area change when there is a dilation of 2 or 3?

Image by Dirk Hünniger
Properties of Surface Area

• When a sphere had a dilation of 2, how much did its surface area increase?
• What about a dilation of 3?
• When a cone had a dilation of 2, how much did its surface area increase?
• What about a dilation of 3?
• When a cylinder had a dilation of 2, how much did its surface area increase?
• What about a dilation of 3?
Properties of Surface Area

• What are the common properties of increasing surface area?
Why does this work?
Properties of Surface Area and Volume

• Do part 3 of your activity packet (calculating the surface area:volume ratio of each shape).
Hypothesis

- For each shape, there is a dilation of 2 and a dilation of 3.
- Make a hypothesis: How will the surface area:volume ratio change when there is a dilation of 2 or 3?

Image by Dirk Hünniger
Properties of Surface Area and Volume

• When a sphere had a dilation of 2, how much did its surface area:volume ratio increase?
• What about a dilation of 3?
• When a cone had a dilation of 2, how much did its surface area:volume ratio increase?
• What about a dilation of 3?
• When a cylinder had a dilation of 2, how much did its surface area:volume ratio increase?
• What about a dilation of 3?
Properties of Surface Area and Volume

• What are the common properties of changing surface area:volume ratio?

Figure 1. A Cubical Critter
Why does this work?
What does this mean for biology?

- Bacteria size
- Tumor size
- Mammal size
- Surface tension for mammals and insects
Do these shapes have any common properties?

What would happen if we assumed that every animal was a perfect cube?

You can do this for some things and still learn a lot about biology.

horse splashesu For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal’s length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving