Modeling Production with a Cubic

One of the models used in economics is a “3 stage production function”. These are cubic polynomials which can be used to represent the production of many different things. At Smith’s Restaurant the number of meals that can be cooked per hour is

\[ M(x) = -0.0615x^3 + 2.116x^2 \]

where \( x \) is the number of cooks.

1. What are the \( x \)-intercepts of \( M(x) \)? Based on the problem, why should it make sense that \((0,0)\) is an \( x \)-intercept?

2. What would be a reasonable domain for \( M(x) \)?

3. Find the average rate of change of \( M(x) \) from:
   
   (a) \( x = 2 \) to \( x = 4 \)

   
   (b) \( x = 8 \) to \( x = 16 \)

   
   (c) \( x = 18 \) to \( x = 22 \)
(d) $x = 22$ to $x = 23$

(e) $x = 27$ to $x = 30$

(f) What does that average rate of change mean for this problem? Explain in words, not in math. Do you see a pattern to the rates of change?

4. Where is the turning point of $M(x)$? What does it mean for Smith’s Restaurant? Explain using words like “cooks” and “meals”, not math!

5. If the owner of the restaurant needs to be able to cook 300 meals per hour, how many cooks should he hire?