Marathon Math Mondays - Week 1

Our standard number system is called *base ten* because of the important role played by the number 10 in our representation of multi-digit numbers. For example

$$\begin{aligned} 4825 &= 4 \times 1000 + 8 \times 100 + 2 \times 10 + 5 \times 1 \\ &= 4 \times 10^3 + 8 \times 10^2 + 2 \times 10^1 + 5 \times 10^0 \end{aligned}$$

We could write numbers in another base. In base five, the number 324 represents the amount

$$3 \times 5^2 + 2 \times 5^1 + 4 \times 5^0$$

the number we normally call 89 in base 10.

Problem 1

Convert the base 2 number 1101 into base 10 to figure out how much it represents.

Problem 2

An unknown number has 3 digits when written in base 5. What are the possible numbers of digits for the same number, written in base 2?

Problem 3

A digit should never be as large asthe base. That's why in base 10 there is no single digit number for 10. Rewrite the base three number 825 using this convention (still in base three).

Marathon Math Mondays - Week 2

This week's assignment is more open ended than last week's. You will need to explore and investigate, and write about your conclusions. *For each problem, you should write in your notebook about your guesses, or "conjectures," how you came up with them, and how you tested them.*

Problem 1

In base 10, the even numbers are easy to spot because they have a 0, 2, 4, 6, or 8 in the one's place. Come up with a similar rule for when a number is even that uses the digits of its base 2 representation. (Hint: Try some examples of numbers you know are even or odd, and convert them to base 2.)

Problem 2

Come up with a rule for when a number is even that uses the digits of its base 4 representation.

Problem 3

Come up with a rule for when a number is even that uses the digits of its base 3 representation. (This rule is tricky!)