Hyperbolic Geometry, Complex Periods, Stereoscopy, and 4D

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2/2/11
Outline

M.C. Escher
  Circle Limits
  Print Gallery

Salvador Dalí
  Harmony of the Spheres
  Crucifixion
M.C. Escher
Wrote *The Elements* in 300 B.C.

All geometry comes down to 5 “postulates”

1. There are line segments
2. There are lines
3. There are circles
4. Right angles are congruent
5. Fishy parallel postulate

Figure: Euclid of Alexandria
Wrote *The Elements* in 300 B.C.

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Figure: Euclid of Alexandria
Euclid’s Fifth Postulate

Figure: If $\alpha + \beta < \pi$, then lines will eventually intersect

- Seems like more of a result than an assumption.
- Spent 2,000 yr.s trying to deduce the 5th from 1–4.
Euclid’s Fifth Postulate

Figure: If $\alpha + \beta < \pi$, then lines will eventually intersect

► Seems like more of a result than an assumption.
► Spent 2,000 yr.s trying to deduce the 5th from 1–4.
In the 1830’s, we realized such a proof was impossible since a “new” geometry satisfied 1–4, but not 5.
“Out of nothing I have created a strange new universe.”
—J. Bolyai commenting on hyperbolic geometry
Figure: “Print Gallery,” 1956
The Droste Effect

Figure: A self-referential picture
Straightened Print Gallery

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Every image that contains a copy of itself has a center.
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Now place the picture in the complex plane with the center at the origin.
The Math

The fact that the picture contains a copy of itself just means that it is invariant under multiplication by a scalar \( r \), which we will call the\textit{ period}. 
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The fact that the picture contains a copy of itself just means that it is invariant under multiplication by a scalar $r$, which we will call the period.

Escher’s twisted picture SHOULD have a complex period. That is, it should be a picture that is invariant under both a rotation and a scaling.
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Figure: Escher’s grid

Figure: computer’s grid
Figure: Loop
Salvador Dalí
Figure: A Holmes-Stereoscope
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Salvador Dalí
Harmony of the Spheres
Crucifixion
Another Way of Seeing 3D
Figure: “The Harmony of the Spheres,” 1978
Figure: Dalí talks with mathematician T. Banchoff in 1975
Figure: “Crucifixion (Corpus Hypercubus),” 1954
Figure: Folding a cross into a cube
Figure: Folding a cross into a cube

Figure: Which of these can be folded into a cube?

Only a, b, and d. In total, there are 11 unfoldings.
**Figure:** Folding a cross into a cube

**Figure:** Which of these can be folded into a cube?

Only a, b, and d. In total, there are 11 unfoldings.
Figure: Folding a hypercross into a hypercube
Figure: Folding a hypercross into a hypercube

Figure: Which of these can be folded into a hypercube?

Only a, c, and e. In total, there are 261 unfoldings.
Figure: Folding a hypercross into a hypercube

Figure: Which of these can be folded into a hypercube?

Only a, c, and e. In total, there are 261 unfoldings.
Figure: The 4D Hypercube