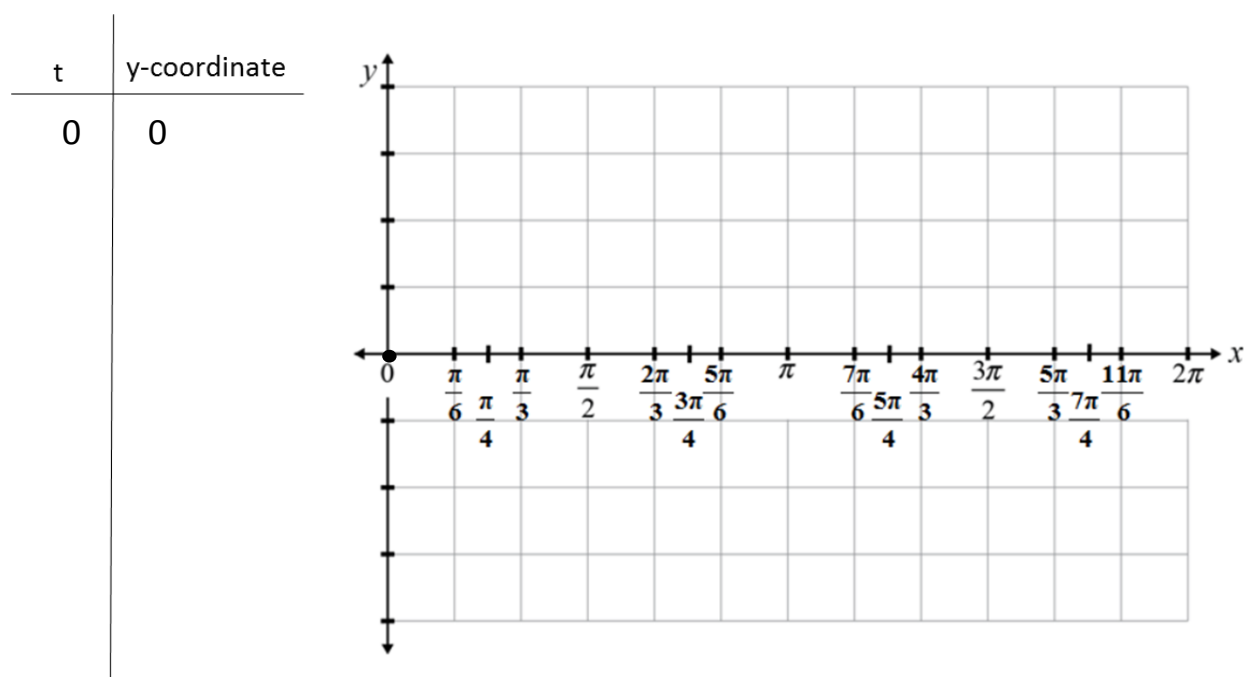


## Graphing Sine and Cosine Group Exploration Activity

Think back to bug on a square. We were able to make a plot of the change in position of the bug as he moves around the square. We can also do this for the bug on a circle. We will plot the  $t$  value on the  $x$ -axis. This represents the distance the bug travels. On the  $y$ -axis, we will either plot the bug's  $x$ -coordinate or the bug's  $y$ -coordinate to represent his position. Because we learned that the  $x$ -coordinate is the cosine and the  $y$ -coordinate is the sine, we will call the resulting graphs either a sine wave or a cosine wave, depending on which position coordinate you are using.

Start at the 0 position and move in the positive direction for all of the points we placed on the unit circle from 0 to  $2\pi$ . This represents one cycle of the wave. List all of the points to be graphed and then plot them on the graph provided. Make sure to number the  $y$ -axis.

### Sine Wave:



The equation for a sine wave generated from a circle of radius one, centered at  $(0, 0)$  is represented by the equation:

$$y = \sin(x)$$

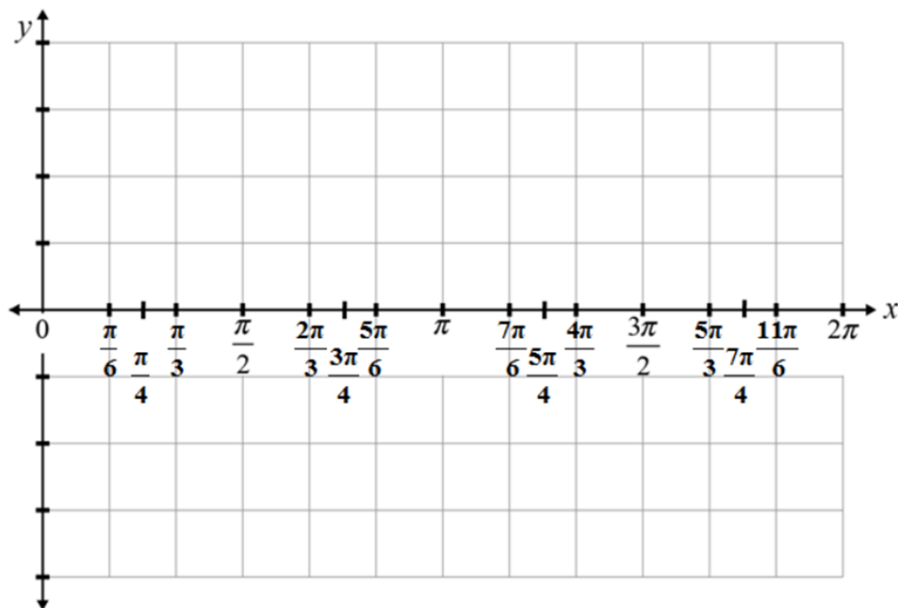
Graph Properties: Identify the following:

- a) Rhythm:
- b) Start:
- c) Maximum:
- d) Minimum:
- e) Zeros:
- f) End:

## Graphing Sine and Cosine Group Exploration Activity

### Cosine Wave:

t	x-coordinate
0	1



The equation for a cosine wave generated from a circle of radius one, centered at (0, 0) is represented by the equation:

$$y = \cos(x)$$

Graph Properties: Identify the following:

- a) Rhythm:
- b) Start:
- c) Maximum:
- d) Minimum:
- e) Zeros:
- f) End:

These graphs represent one cycle of the wave. If our bug continued to go around the circle, or went in the negative direction around the circle, the cycle would repeat infinitely in either direction. Sine and cosine waves are generated from repetitive motion. Also, the t value for this represents the distance traveled, but it can also be time it took to complete a cycle.

## Graphing Sine and Cosine Group Exploration Activity

### 1. Amplitude

Using your calculator, graph the curve which results from each function:

For a) and b) graph the parent function  $y = \sin(x)$  as well for comparison. For c) and d) graph  $y = \cos(x)$ .

a)  $y = 3 \sin(x)$

b)  $y = 4 \sin(x)$

c)  $y = 5 \cos(x)$

d)  $y = 2 \cos(x)$

Describe what is changing between the graphs and how it relates to the equations.

**Amplitude is defined as the maximum height of the wave from the midline.**

Write the equation for the sine and cosine waves to include amplitude. Represent the amplitude with the variable  $a$ .

Because amplitude is a maximum, it can only be positive. Therefore:

$$\text{Amplitude} = |a|$$

Using your calculator, graph the curve which results from each function. Graph each pair on the same plot. Also, state the amplitude of each equation:

a)  $y = 3 \sin x$ ,  $y = -3 \sin x$

b)  $y = 4 \sin x$ ,  $y = -4 \sin x$

c)  $y = 3 \cos x$ ,  $y = -3 \cos x$

d)  $y = 2 \cos x$ ,  $y = -2 \cos x$

Describe what is changing when a negative sign is in front of the amplitude:

# Graphing Sine and Cosine Group Exploration Activity

Practice:

1) Graph  $y = 2\sin(x)$

a) Rhythm:

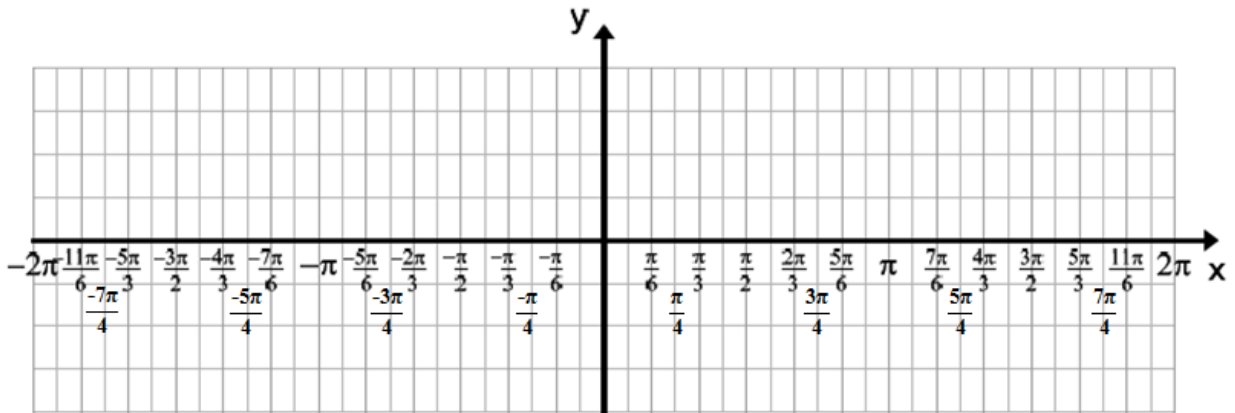
b) Start:

c) Maximum:

d) Minimum:

e) Zeros:

f) End:



2) Graph  $y = -3\cos(x)$

a) Rhythm:

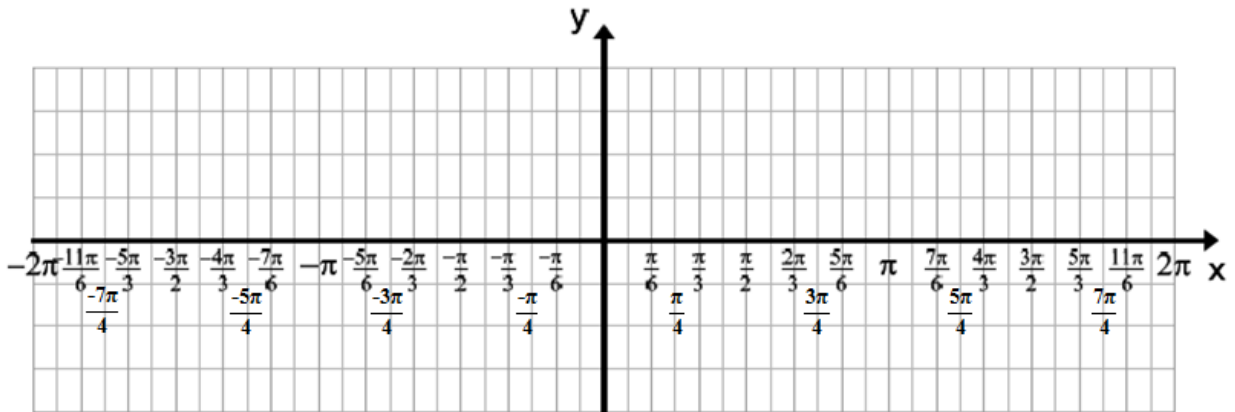
b) Start:

c) Maximum:

d) Minimum:

e) Zeros:

f) End:



# Graphing Sine and Cosine Group Exploration Activity

## 2. Frequency and Period

Using your calculator, graph the curve which results from each function:

a)  $y = \sin(2x)$

b)  $y = \sin(3x)$

c)  $y = \cos(3x)$

c)  $y = \cos(4x)$

Describe in words the difference between these graphs and how it relates to the equations.

**The frequency is defined as the number of cycles that occur per unit of distance or time. It is what is changing in the equations above.**

Re-graph some of the equations above and count to see how many cycles you see in one unit to confirm that you understand what is changing. It might help to graph the parent function  $y = \sin(x)$  or  $y = \cos(x)$  along with each graph.

Write the equation for the sine and cosine waves to include frequency. Represent the frequency with the variable  $b$ .

**The period of a sine or cosine function is calculated by dividing  $2\pi$  (the standard width of one cycle) by the frequency of the trigonometric function. The period of a trigonometric function represents the width of one cycle of the curve. Period is crucial to know when you are graphing with paper and pencil. It is less crucial to calculator-assisted graphs.**

Give the frequency, period and amplitude of each of the following functions:

a)  $y = 3 \sin(2x)$

b)  $y = -6 \cos(4x)$

c)  $y = 4 \sin(\frac{1}{2} x)$

Frequency:

Frequency:

Frequency:

Period:

Period:

Period:

Amplitude:

Amplitude:

Amplitude:

## Graphing Sine and Cosine Group Exploration Activity

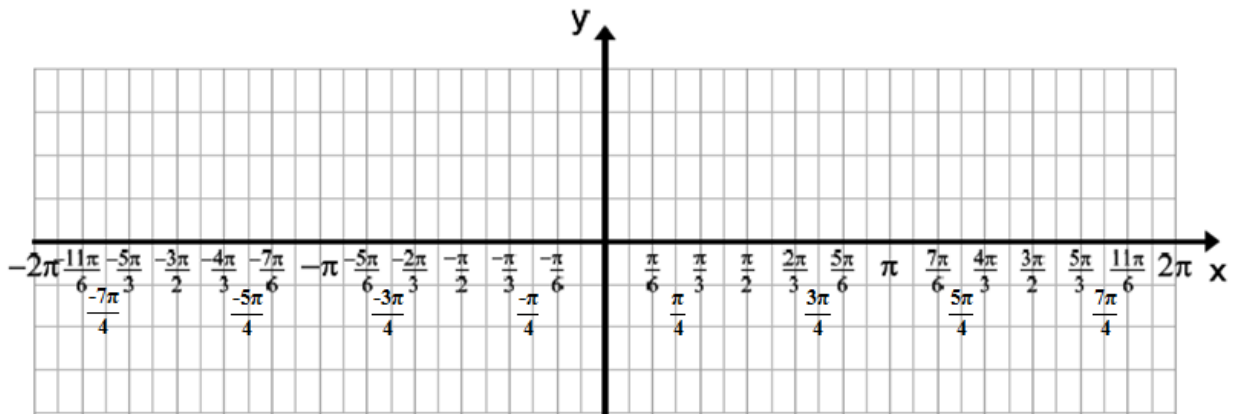
Practice:

1) Graph by hand  $y = -\frac{1}{2}\sin(2x)$

Amplitude: \_\_\_\_\_ Rhythm: \_\_\_\_\_

Period: \_\_\_\_\_ Quarter Period: \_\_\_\_\_

Start: \_\_\_\_\_ End: \_\_\_\_\_

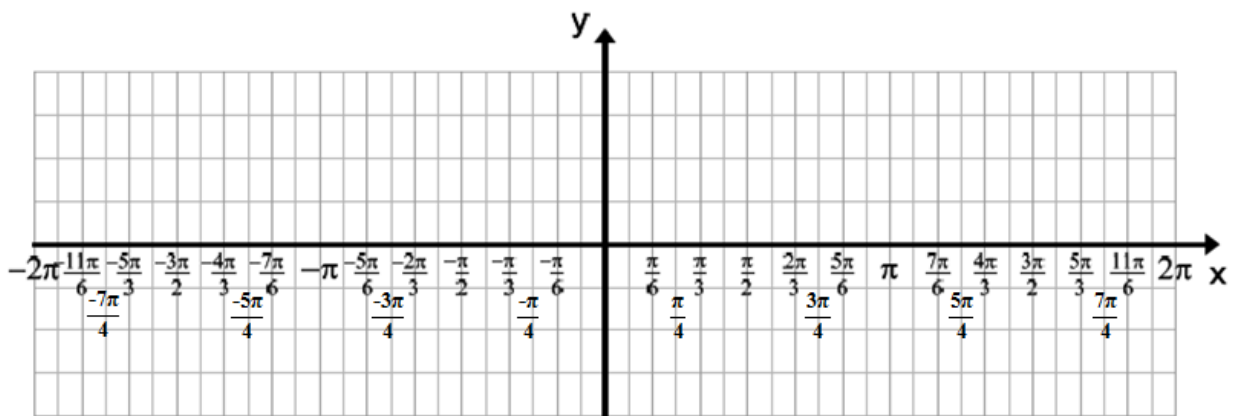


2) Graph by hand  $y = 4\cos\left(\frac{x}{2}\right)$

Amplitude: \_\_\_\_\_ Rhythm: \_\_\_\_\_

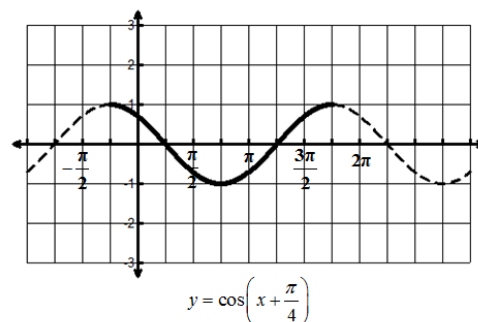
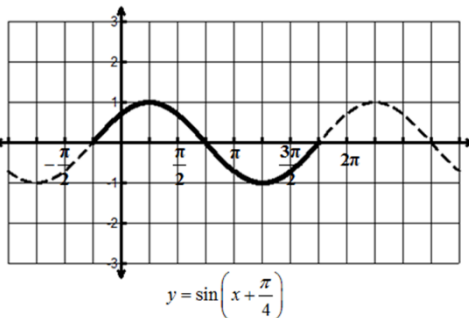
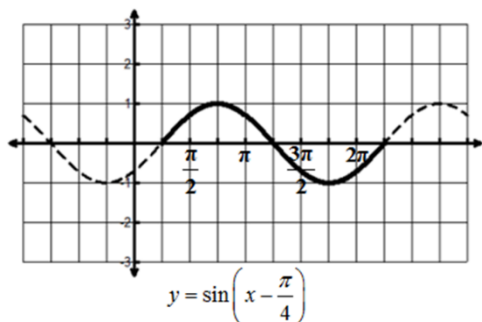
Period: \_\_\_\_\_ Quarter Period: \_\_\_\_\_

Start: \_\_\_\_\_ End: \_\_\_\_\_

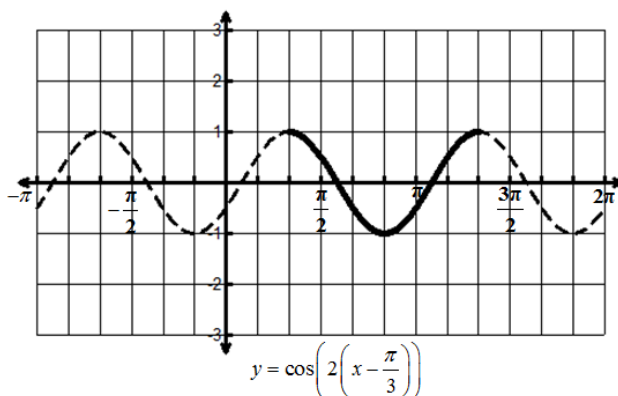
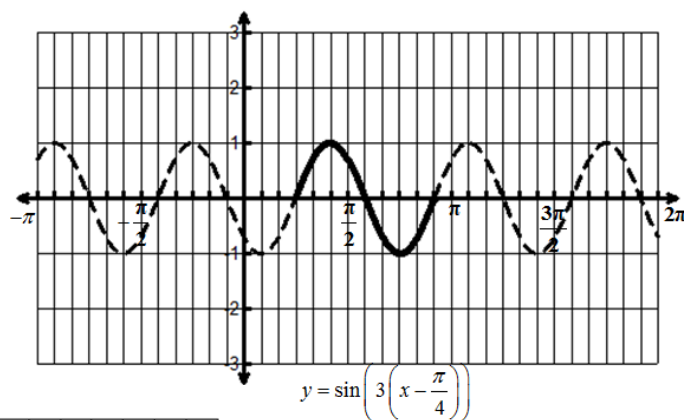
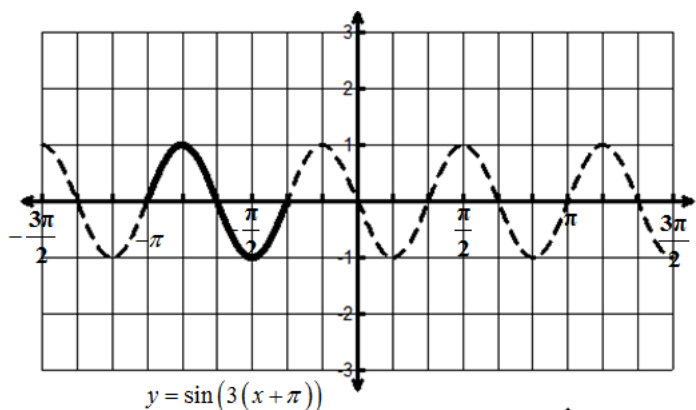


## Graphing Sine and Cosine Group Exploration Activity

### 4. Phase Shift



Examine the graphs and their equations above. Describe what is changing in each graph and how it relates to the equations.



Describe how these graphs and equations are different than the first examples.

The general equation for sine or cosine including a shift in phase can be represented as below:

$$y = a\sin(b(x + c)) \quad \text{or} \quad y = a\cos(b(x + c))$$

## Graphing Sine and Cosine Group Exploration Activity

Sketch the curve which results from graphing the following functions. Draw a solid line for the first cycle of the curve, and a dotted line for the full  $(-2\pi, 2\pi)$  range like in the graphs above. Make sure to determine all of the important information/points.

a)  $y = \cos\left(2\left(x + \frac{\pi}{3}\right)\right)$

Amplitude:

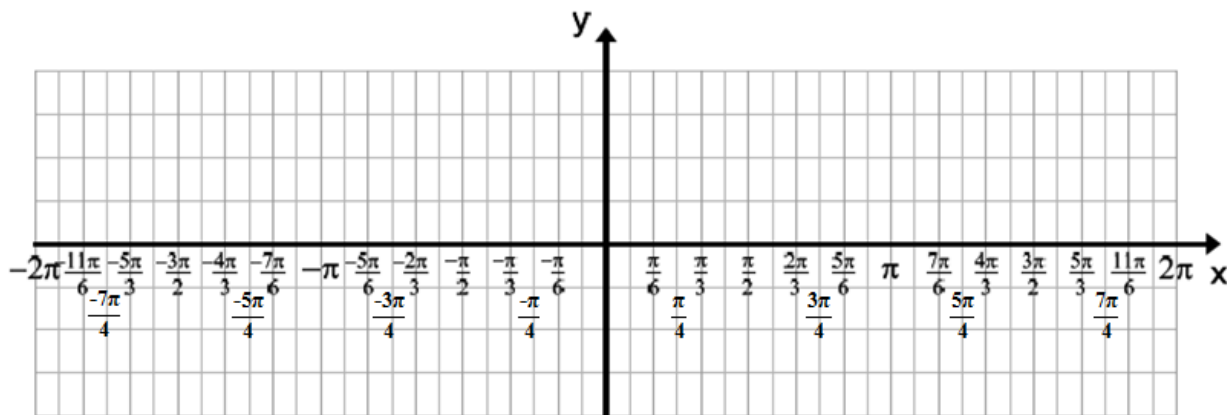
Rhythm:

Period:

Quarter Period:

Start (Horizontal Shift):

End:



b)  $y = 2\sin\left(3\left(x + \frac{\pi}{4}\right)\right)$

Amplitude:

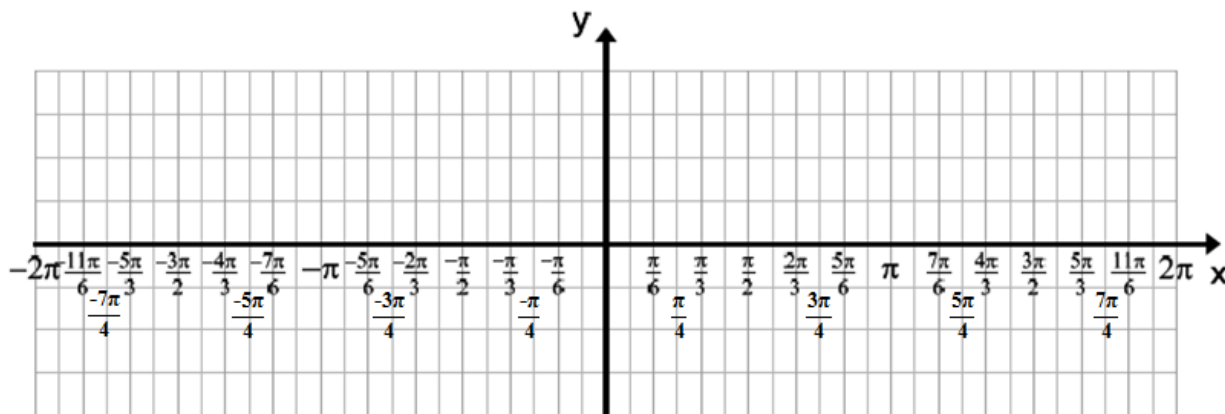
Rhythm:

Period:

Quarter Period:

Start (Horizontal Shift):

End:





## Graphing Sine and Cosine Group Exploration Activity

c)  $y = 3\cos\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$

Amplitude:

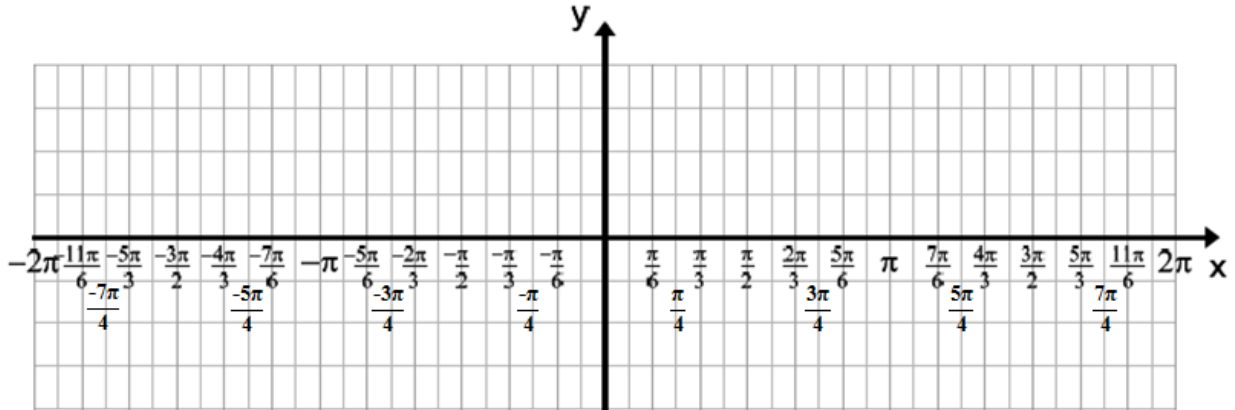
Period:

Period:

Quarter Period:

Start (Horizontal Shift):

End:



### EXTRA

Can you describe the relationship between  $y = \sin x$  and  $y = \cos x$  which is exhibited by the following functions?

Graph each pair of functions using your calculator and compare the resulting curves:

a)  $y = 2 \sin x$ ,  $y = 2 \cos(x - \pi/2)$

b)  $y = 3 \cos x$ ,  $y = 3 \sin(x - \pi/2)$

## Graphing Sine and Cosine Group Exploration Activity

### 5. Vertical Shift

Using your calculator, graph the curve which results from each function:

a)  $y = \sin(x) + 1$

b)  $y = \cos(x) + 3$

c)  $y = \cos(x) - 2$

d)  $y = \sin(x) - 4$

Summarize how a change in the constant affects the graph produced.

The general equation for a sine wave including a vertical shift can be written as follows:

$$y = a\sin(b(x + c)) + d \quad \text{or} \quad y = a\cos(b(x + c)) + d$$

Sketch the curve which results from graphing the following function. Provide the values for the important information/points of the graph:

1)  $y = 2\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 1$

Amplitude:

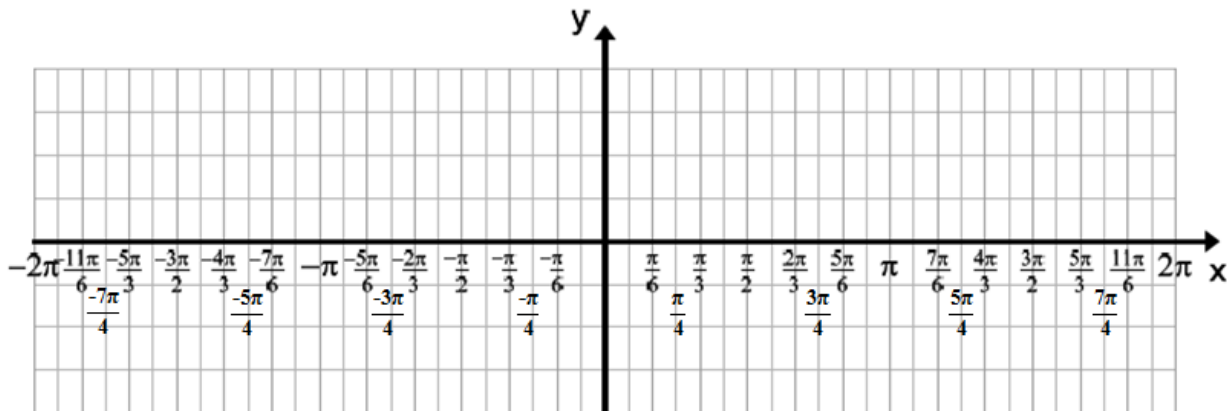
Period:

Period:

Quarter Period:

Start (Horizontal Shift):

Vertical Shift:



## Graphing Sine and Cosine Group Exploration Activity

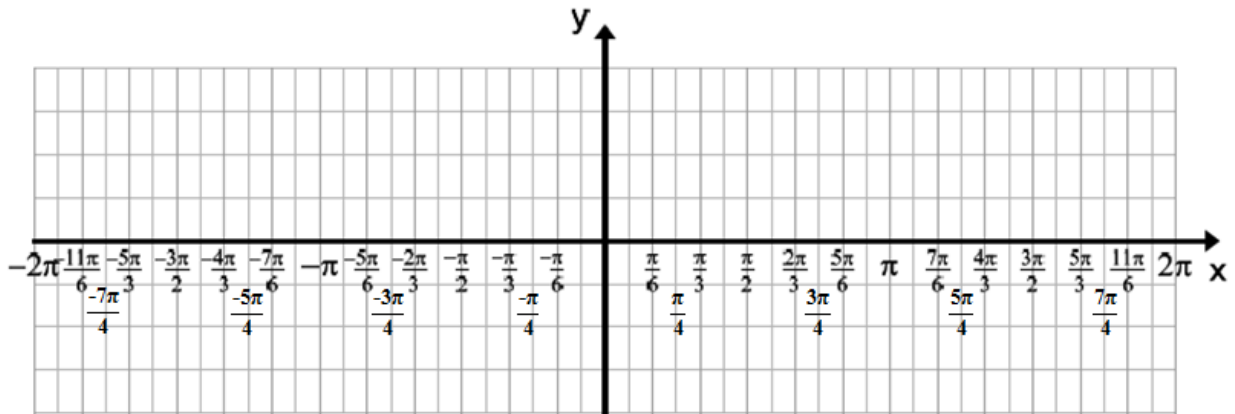
1)  $y = -2\cos\left(\frac{1}{2}\left(x - \frac{\pi}{6}\right)\right) - 2$

Amplitude: \_\_\_\_\_ Rhythm: \_\_\_\_\_

Period: \_\_\_\_\_ Quarter Period: \_\_\_\_\_

Start (Horizontal Shift): \_\_\_\_\_

Vertical Shift: \_\_\_\_\_



### 6. Summary

Given the following trigonometric function:

$$y = 3 \sin 4(x + \pi/4) + 1$$

write, in your own words, what would happen to the graph of this function when:

a) the AMPLITUDE, a, is changed from 3 to 4

e)  $(x + \pi/4)$  is changed to  $(x + \pi/3)$

b) the AMPLITUDE, a, is changed from 3 to -2

f)  $(x + \pi/4)$  is changed to  $(x - \pi/3)$

c) the FREQUENCY, b, is changed from 4 to 2

g) the CONSTANT, d, is changed from 1 to 3

d) the FREQUENCY, b, is changed from 4 to 8

h) the CONSTANT, d, is changed from 1 to -2