Using Manipulatives in the Classroom

I. Base Blocks
The Math department has base three, base four, base five, base six, and base ten blocks.

- singles (small cubes)
- longs
- flats
- (big) cubes

Counting and place value

Addition

Subtraction

Multiplication

Division

Fractions and decimals
Counting, place value, and decimals:

What comes before $30_5$?

What comes after $44_5$?

Why do we call $10^2$ "ten squared"?

If $\square$ is 1, what is $\square$?

If □ is 0.01, what is 1?
Addition and Subtraction:

\[24_5 + 33_5 = \text{Use regrouping.}\]

\[32 - 27 = \text{Use regrouping (exchanging).}\]

\[32 - 37 = \text{Use zero pairs.}\]

Compensation: increase both numbers by the same amount; the difference remains the same.

\[7 - (-5) = 12\]
Multiplication:

\[13_5 \times 24_5 = \]

Set up frame; regroup results as needed (depends on base)

Link to area model

View as discrete model --> link to fraction multiplication

For example, the diagram for \(13_5 \times 24_5\) could also model \(1 \frac{3}{5} \times 2 \frac{4}{5}\). Note that "1" inside the frame (after multiplying) is one square; multiplication gives you square units. The answer is \(2 + \frac{2}{5} + \frac{2}{25}\), or \(2 \frac{12}{25}\).

Link to algebra: algebra tiles
Algebra Tiles

- singles (constant terms)
- unknown bars (linear terms)
- unknown squares (quadratic terms)

The flip sides are another color to represent negative quantities.

Represent the following as an algebraic equation, and solve:

- \( x + 1 = 4 \)
  - \( x = 3 \)

Solve for \( x \) using tiles: \( 2x - 3 = 5 \)

- add 3 to both sides:
- then partition:
  - \( x = 4 \)
Multiply using tiles:
\[(x + 1)(2x - 3)\]
\[= x^2 + x^2 + x + x - x - x - x + 3\]
\[= 2x^2 - x - 3.\]

Challenge: how to explain convincingly that a positive times a negative is negative, and a negative times a negative is positive. (Suggestions welcome!)
Geoboards

polygons or
(pseudo)circles

Shape, area and perimeter:

Find the area of a trapezoid with bases of length 2 and 3 and height 1.

Make two more shapes with the same area as the trapezoid.

Do all three shapes have the same perimeter?

Note--students may focus on the number of pegs enclosed rather than the area enclosed.
Fractions:
Show $\frac{1}{3}$ of in two different ways.

If is $\frac{1}{4}$, what is 1?

Again, the focus should be on the area enclosed.

Can also use geoboards to introduce irrational numbers (recall Pythagoras).
Pattern Blocks

Six shapes per set:
- equilateral triangle (green)
- rhombus (blue)
- trapezoid (red)
- hexagon (yellow)
- square (orange)
- small rhombus (tan)

Patterns, symmetry, tessellations:

Make a pattern with bilateral symmetry.

Make a pattern with three point radial symmetry.

Which of the pattern blocks can tessellate (by themselves)?
Why? What happens if you change the shape slightly (try drawing pictures)?

Make a tessellation using two shapes.
Fractions:

If $\frac{3}{4}$, what is the value of $\frac{2}{4}$?

2/4.

If two hexagons equal 1 1/5, show 1.

--Link to multiplication and division of fractions.

Pattern blocks can catch the interest and attention of kids who are otherwise not engaged in the math class. Let them be creative.
Other resources:

National Library of Virtual Manipulatives
  nlvm.usu.edu/en/nav/vlibrary.html

Web search on "mathematical manipulatives"

Math 302A/B instructors

  Feel free to email questions to me anytime:
  Brenae Bailey
  bbailey@math.arizona.edu

I'll do my best to answer them!