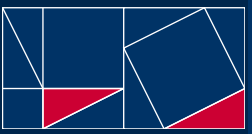
Lemonade Stands
Plan for a Making Connections Workshop

Arizona team: Josh Chesler, J.P. Cossey, Melinda Jensen, Alyssa Keri, William McCallum

Institute for Mathematics and Education

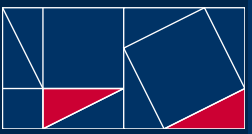
June, 2008



INSTITUTE FOR MATHEMATICS & EDUCATION

Manipulations on expressions and equations

The key mathematical idea that we want to be brought out by this problem is the idea that the rules governing what you can do to expressions are different from the rules governing what you can do to equations.



INSTITUTE FOR MATHEMATICS & EDUCATION

What you can do to expressions

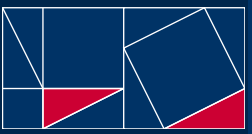
- The rules governing what you can do to expressions are grounded in the basic nine rules of arithmetic—four rules for each operation (commutativity, associativity, existence of identity and inverse) and the distributive law linking the two operations.
- These rules underpin many of the operations of algebra in a disguised way, such as cancelling a factor from top and bottom of a fraction (this is quite an involved deduction from the axioms, starting with the definition $a/b = ab^{-1}$) and collecting like terms (distributive law).

What you can do to equations

The rules governing what you can do to equations are different. An equation is a statement of equality between two expressions. In addition to manipulating the expressions on either side according to the rules for expressions, we can perform the same (reversible) operation on both sides of the equation.

Other issues

Other mathematical issues include the difference between expressions and equations, and the difference between evaluating expressions and solving equations.
The specific domain in which we deal with these issues is linear expressions and equations, and interpreting the slope and y-intercept.



INSTITUTE FOR MATHEMATICS & EDUCATION

The Problem

Melinda and JP are selling lemonade at a stand in her front yard. The linear expression below models her total profit in dollars after h hours.

$$\frac{2}{3}(6h - 5) + 4h.$$

1. What is her profit after 2 hours? Explain.
2. When will her profit be \$30? Explain.
3. How much does her profit increase each hour? Explain.
4. Evaluate the expression for $h = 0$. What does this value mean in the context of the problem?
5. Explain how the answer to part 3 could be calculated mentally just by looking at the expression.
6. Explain how the answer to part 4 could be calculated mentally just by looking at the expression.

Question design

The questions were designed to elicit student thinking in working with expressions and equations. Of particular interest is simplifying expressions vs. solving equations and whether reflecting on the process reduces the error rate. Parts 5 & 6 were designed to encourage students to use strategic thinking, developing their ability to think ahead in the calculation without having to perform the calculation. Included is the understanding that different forms of an expression provide different information about the context of the problem, and understanding which form is most efficient for a given problem.

Common errors

The anticipated common errors are:

- Using steps that create non-equivalent expressions & equations
  - Clearing fractions in expressions
  - Attempting to clear the fraction, but instead multiplying the numerator by the value of the denominator and then drop the denominator
  - Attempting to clear the fraction, but including multiplying the denominator inside the parenthesis
  - Confusing combining like terms with inverse operations that would be used to solve equations
  - Distribution errors

Common Errors (cont.)

- When calculating profit increase
  - They may try to read one value from the given expression (2/3 or 6 or 4).
  - Incorrectly calculating slope between two points because of fraction operation errors
- When calculating the y-intercept
  - They may try to read one value from the given expression (-5).

The Back Story

JP and Melinda work out a deal with their parents to start a Lemonade Stand franchise. JP's parents will cover the start up costs for the stand, and selling in his neighborhood he can make $4 an hour. Melinda lives in a lemonade-starved dangerous neighborhood. To get started she needs to do business with the local (10-speed) bicycle gang, the Rattlesnake Boys, who will provide protection for her stand. She works out a deal which gives one-third of her local stand to the Rattlesnake Boys, start up fees and profits included. If she needs to buy $5 in lemons and makes $6 an hour in her neighborhood- how much profit does the Lemonade franchise make after h hours?

Professional Development Activity

- Before meeting, give packet with
  - Problems 1, 2, 9 from this year's problem packet and the lemonade stand problem
  - The samples of student work EAZ-10-0508 and EAZ-22-0508 for Problem 2; CAZ100508 for Problem 9; and CAZ120508 for Problem 1 (contact the Institute for these samples)

  Ask teachers to do the problems and look at the student work in advance of the meeting.
- Teachers meet and discuss (see discussion questions)
- Teachers give lemonade problem to their students
- Teachers meet again, discuss student work, develop a lesson based on it. Assess what student confusions are about expressions and equations. Decide what issues need to be addressed, what the teacher needs to do differently.
- Could use with a beginning Algebra II year or an ending Algebra I year.

Discussion Questions

- Is the error in Problem 9, CAZ100508 a procedural error or a conceptual error?
- What are the similarities and differences between Problem 2, EAZ220508 and EAZ100508?
- Why doesn't the student's method for part (b) in Problem 1, CAZ120508 transfer to part (d).
- What mathematical ideas are presented in the different parts of the lemonade stand problem.
- What kind of mistakes do you think students will make?
- What is the mathematical justification for cancelling from top and bottom of a fraction?
- What mathematics to students need to be able to do in their head?
- What can you get from asking students to explain their work that you can't get from asking them to show their work?
- How are the mathematical concepts integrated?

$$\frac{5x+10}{5}$$

Common, but bad practise, will cross out numbers and put 1s and 2s. If they've thought more about the math, they might do, in Algebra I

$$\frac{5x}{5}+\frac{10}{5}$$

or, in Algebra II

$$\frac{5(x+2)}{5}$$

Student mistakes

$$\frac{4(x+8)}{2}=2(x+4)$$

$$\frac{5x+10}{5}=x+10$$

$$\frac{(x+2)(x+2)}{4}=\frac{1}{4}$$