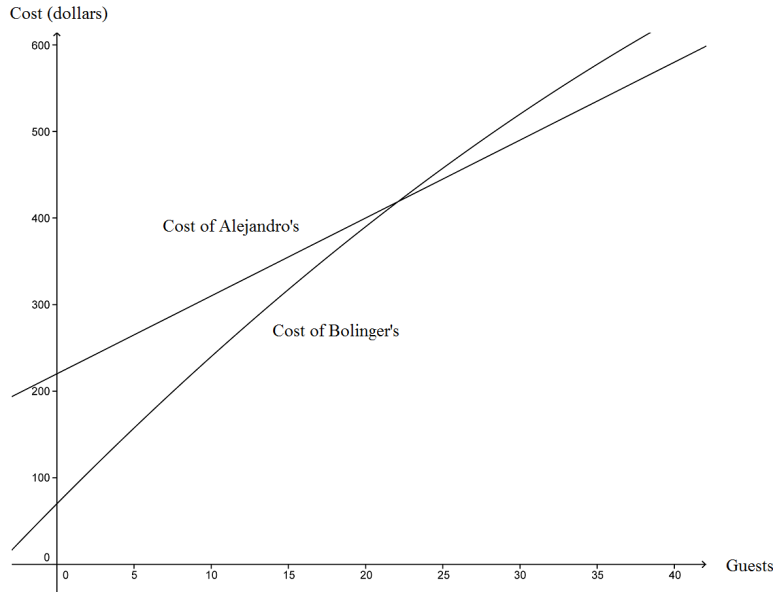


# Workshop on the Transition from High School to College

## The Calculus Problem Solver: Functions

1. Janine wants to host a party for her friends at one of her favorite restaurants. At Alejandro's, the cost of hosting a party is a linear function of the number of guests. At Bolinger's, the cost of hosting a party is a quadratic function of the number of guests. The graphs of both functions are given in the figure below.



- (a) If Janine can afford to spend \$500, how many guests can she afford to host at Alejandro's?
- (b) For approximately what number of guests do the two locations cost the same amount of money?
- (c) For approximately what number of guests does Alejandro's cost twice as much as Bolinger's?
- (d) The graph of the cost function for Alejandro's is a straight line. Estimate the slope of this line, and describe in real-world terms what the slope represents.
- (e) Janine decides to host her party at Bolinger's and ends up paying an average of \$20 per guest. Use the graph to determine how many guests Janine had the party. Explain your approach.

2. Given below are six descriptions of classes of functions. For each description, decide which class of functions (if any) is being described, and indicate your choice with the appropriate letter. Your choices are

**E** – Exponential functions of the form  $f(t) = Ab^t$ , where  $A > 0$  and  $b > 0$ .

**S** – Sinusoidal functions of the form  $f(t) = y_0 + A \sin(c(t - t_0))$ , where  $A \neq 0$ .

**P** – Nonzero polynomial functions of the form  $f(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0$ .

**N** – None of the above.

Write your answer to the left of each part. You do not need to explain your answers.

- (a) These functions are periodic.
- (b) These functions have vertical asymptotes.
- (c) The range of one of these functions may be  $[5, 17]$ .
- (d) These functions are always invertible.
- (e) One of these functions may have exactly nine zeroes.
- (f) One of these functions may be decreasing on  $(-\infty, 3]$  and increasing on  $[12, \infty)$ .

8. (3 points each) Multiple choice. Circle *the* correct answer to each one of the following questions. (No partial credit!)

*Note: any angle measures appearing in this problem are given in radians.*

- (I) The equation  $-3 \tan x + 5 = 1$  has exactly  $n$  solutions on the interval  $-\pi/2 \leq x \leq 2\pi$ .
- (a)  $n = 1$
  - (b)  $n = 2$
  - (c)  $n = 5$
  - (d)  $n = 6$
- (II) Find all solutions to the equation  $3^x = x^3$ .
- (a)  $x = 3$
  - (b)  $x \simeq 2.478$
  - (c)  $x = 3, x \simeq 2.478$
  - (d) No solutions exist.
- (III) Which of the following is true of the function  $f(x) = \frac{16}{x^2 - 9} + 2$ ?
- (a)  $f(x) = f(-x)$
  - (b)  $-f(x) = f(-x)$
  - (c) It is neither odd nor even.
  - (d) It is a quadratic function.
- (IV) The graph of a function  $g(x)$  is horizontally stretched, then flipped over the  $x$ -axis and finally shifted up  $k$  units. Which of the following is the formula for the resulting function?
- (a)  $-2g(x) + k$
  - (b)  $0.5g(-x) + k$
  - (c)  $-g(2x) + k$
  - (d)  $-g(0.5x) + k$
- (V) You just won a 5.0 mega-pixel digital camera for participating in an online promotion. According to a web review this particular model is currently valued at 500 dollars and depreciates according to the formula  $C = 500(0.8)^t$  per year (since 2005.) Which of the following is true?
- (a) The camera loses value at a rate of 1.8% per month.
  - (b) The camera loses value at a rate of 80% per year.
  - (c) The camera loses value at a rate of exactly 10% every six months.
  - (d) The camera loses value at a continuous rate of 20% per year.

2. (17 points) In order to gain popularity among students, a brand new on-campus hair salon plans to offer a special promotion. The cost of a haircut, in dollars, at the salon as a function of time, in days since February 10th may be described as

$$C(t) = \begin{cases} 9, & 0 \leq t \leq 3 \\ 9 + t, & 3 < t \leq 8 \\ 20, & 8 < t < 28 \end{cases} .$$

(Assume  $t$  only takes whole number values.)

- (a) (3 pts.) If you want them to give them a try, on what date(s) should you have a haircut in order to get the best price?
- (b) (2 pts.) How much will a haircut cost on Feb. 18th?
- (c) (2 pts.) On what date will a haircut cost 13 dollars?
- (d) (3 pts.) The cost of a haircut is at least  $A$  dollars  $B$  days into the promotion. Write an expression that describes this sentence using function notation and mathematical symbols only.
- (e) (4 pts.) Calculate  $C(9) - C(8)$  and interpret its meaning in the context of the problem.

*(This problem continues on the next page.)*

(This is a continuation of Problem 2. For your convenience, the original problem statement is reprinted here.)

In order to gain popularity among students, a brand new on-campus hair salon plans to offer a special promotion. The cost of a haircut, in dollars, at the salon as a function of time, in days since February 10th may be described as

$$C(t) = \begin{cases} 9, & 0 \leq t \leq 3 \\ 9 + t, & 3 < t \leq 8 \\ 20, & 8 < t < 28 \end{cases} .$$

(Assume  $t$  only takes whole number values.)

- (f) (3 pts.) On average, the cost of a haircut goes up about 85 cents per day during the first two weeks of the promotion period. Which of the following expressions best describes this statement? No explanation is necessary.

(I)  $\frac{C(13) + C(0)}{2}$

(II)  $\frac{C(13) - C(0)}{13}$

(III)  $\frac{\Delta C}{\Delta t}$

(IV)  $\frac{C(13)}{13}$

(V)  $\frac{C(\text{Feb. 23.}) - C(\text{Feb. 10})}{13}$

3. (2 points each) Circle “TRUE” or “FALSE” for each of the following problems. Circle “TRUE” only if the statement is *always* true. No explanation is necessary.

(a) If two variables  $x$  and  $y$  are related by  $3x + 5 = -2y + 7$ , then  $x$  is a function of  $y$  and  $y$  is a function of  $x$ .

TRUE      FALSE

(b) An increasing function has a positive rate of change over every interval.

TRUE      FALSE

(c) The domain of the function  $y = \frac{1}{x^2 - 25}$  is all real numbers  $x$  excluding 5.

TRUE      FALSE

(d) If  $a > 0$ , the range of the function  $k(s) = a(0.2)^s$  is  $k(s) \geq 0$ .

TRUE      FALSE

(e) An exponential function of the form  $t(x) = b^x$  is always concave up.

TRUE      FALSE

4. (12 pts.) Consider the formulas and tables given below. Assume  $a$ ,  $b$  and  $c$  are all positive constants.

(A)  $\frac{x}{y} \parallel \begin{array}{cccc} 0 & 40 & 80 & 160 \\ 2.2 & 2.2 & 2.2 & 2.2 \end{array}$       (B)  $y = ax^{23}$       (C)  $\frac{x}{y} \parallel \begin{array}{cccc} -8 & -4 & 0 & 8 \\ 51 & 62 & 73 & 95 \end{array}$

(D)  $\frac{x}{y} \parallel \begin{array}{cccc} 3 & 4 & 5 & 6 \\ 18 & 9 & 4.5 & 2.25 \end{array}$       (E)  $y = \frac{-1}{5^{-x+2}}$       (F)  $\frac{x}{y} \parallel \begin{array}{cccc} -4 & -3 & 4 & 6 \\ 18 & 0 & 4.5 & -2.25 \end{array}$

(G)  $y = \sqrt{x^4 + 16}$       (H)  $cy = a^3b^x$       (I)  $y + ax = bx^2 + cx$

In the spaces provided below, list all of the equations and tables (A)–(I) above which satisfy each one of the following statements. No explanation is necessary.

- These (and only these) could represent linear functions:\_\_\_\_\_
- These (and only these) could represent quadratic functions:\_\_\_\_\_
- These (and only these) could represent exponential functions:\_\_\_\_\_

6. (13 pts.) In the magical world of Harry Potter the Weasley brothers, Fred and George, invent “extendable ears,” a type of string which allows them to eavesdrop on adult conversations.

The length of this magical string, grows or shrinks depending on the distance  $d$  between the Weasley brothers and the source of the conversation they want to hear.

When not in use, the “extendable ears” are 1 foot long. When put to work, their length grows 4 feet for every 3 yards separating the brothers from the source of the sound.

- (a) (2 pts.) In a sentence, indicate what in this verbal description of the “extendable ears,” tells you that their length,  $l$ , is a linear function of  $d$ .

- (b) (4 pts.) Write down the linear equation giving the length of the “extendable ears,”  $l$ , as a function of the distance  $d$  between the Weasley brothers and the source of sound.

- (c) (3 pts.) How long do the “extendable ears” get when the source of voices is 69 yards away from the brothers? Show step-by-step work.

- (d) (4 pts.) Sometimes Fred and George can estimate the location of a certain voice, judging by the length of the “extendable ears.” Write a linear equation that gives you the distance,  $d$ , in yards, from the sound source, as a function of the length,  $l$ , in feet, of the extendable ears. Show step-by-step work.