Idea of Approximation

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Occurrences

1. Approximation of functions
   a) Linearization
   b) Taylor polynomials
2. Approximations of geometrical measures
   a) Area, arclength,surface area, volume
   b) Tangent to a curve
3. Approximation of a definite integral (Riemann sum - Includes 2a)
4. Approximation of numerical solutions to equations

Language: Approximation

- Find a specific numerical approximation to a number; ideally with an error bound (e.g. linear approximation of $\sqrt{4.1}$)
- A limiting process that in principle can approximate a quantity with an arbitrary small error (the Riemann sums to approximate the area under the graph of a function).

Ways of Understanding

Example of linearization problem

Approximate the value of $\sqrt{4.1}$

Conceptual Difficulties:

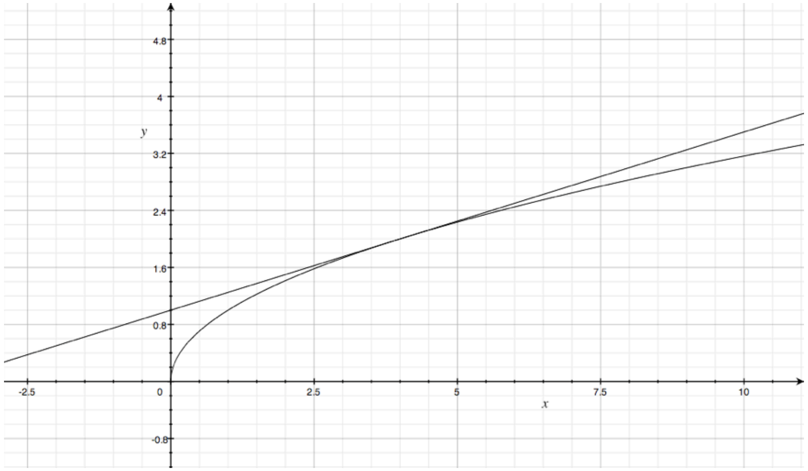
- Recognize that this problem could be solved by linearization.
- Identify this number as f(x) for an appropriate function and an appropriate value of x. (The function is not unique).
- Appropriate for x is x_0+h.

What does it means understanding what linearization means?

- Good understanding of the derivative,

$$f'(x) \approx \frac{f(x) - f(x_0)}{x - x_0}$$

- Good understanding of the geometry of the graph and the tangent line at the point.



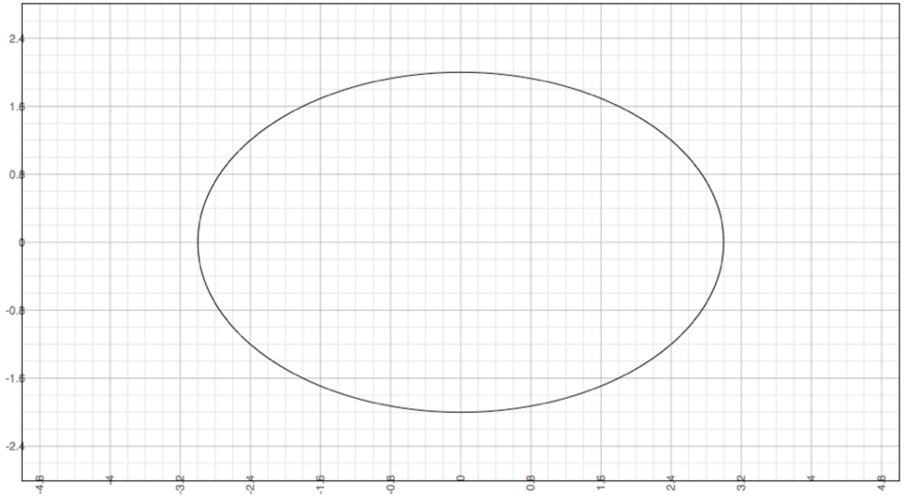
Why giving the problem of $\sqrt{4.1}$ using linearization?

May be a better problem is to tell the students if you know that f(4) = 2 and the derivative of f, f'(4)= 1/4, approximate the value of f(4.1).

Riemann Sums

1- Approximate the area of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$



2 - Approximate the area under the graph of f(x) for x in [0,1] and n=20.

- Notation nightmare
- Translating the picture to the algebraic computations
- Upper, lower sums, errors
- Recognizing an approximation (e.g. arclength) as a Riemann sum.