Idea of Approximation

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1. Approximation of functions
   a) Linearization
   b) Taylor polynomials
2. Approximations of geometrical measures
   a) Area, arclength, surface area, volume
   b) Tangent to a curve
3. Approximation of a definite integral
   (Riemann sum - Includes 2a)
4. Approximation of numerical solutions to equations
Language: Approximation

- Find a specific numerical approximation to a number; ideally with an error bound (e.g. linear approximation of $\sqrt{4.1}$)
- A limiting process that in principle can approximate a quantity with an arbitrary small error (the Riemann sums to approximate the area under the graph of a function).
Ways of Understanding

Example of linearization problem

Approximate the value of $\sqrt{4.1}$

Conceptual Difficulties:

• Recognize that this problem could be solved by linearization.

• Identify this number as $f(x)$ for an appropriate function and an appropriate value of $x$. (The function is not unique).

• Appropriate for $x$ is $x_0 + h$. 
What does it means understanding what linearization means?

• Good understanding of the derivative,

\[ f'(x) \approx \frac{f(x) - f(x_0)}{x - x_0} \]

• Good understanding of the geometry of the graph and the tangent line at the point.
Why giving the problem of \( \sqrt{4.1} \) using linearization?

May be a better problem is to tell the students if you know that \( f(4) = 2 \) and the derivative of \( f \), \( f'(4) = 1/4 \), approximate the value of \( f(4.1) \).
Riemann Sums

1- Approximate the area of the ellipse

\[ \frac{x^2}{9} + \frac{y^2}{4} = 1 \]
2 - Approximate the area under the graph of $f(x)$ for $x$ in $[0,1]$ and $n=20$.

- Notation nightmare
- Translating the picture to the algebraic computations
- Upper, lower sums, errors
- Recognizing an approximation (e.g. arclength) as a Riemann sum.