

Sample Exam A
Master of Arts in Teaching (MAT) Masters Exam – Spring 2006
UNL Department of Mathematics

The MAT exam consists of a (two-part) written section and an oral presentation to an MAT faculty committee. The written part of your exam (given below) should be submitted on or before Friday, April 7, 2006. Your oral presentation is scheduled for Friday, April 14, 2006.

Part IA. Below find three questions that your Math in the Middle coursework should prepare you to answer. It is anticipated that taken together, your answers will be about 5 to 8 pages.

1. Understanding irrational numbers

- a. Given a careful proof that between any two real numbers there is a rational number and an irrational number and build on this fact to argue that between any two real numbers there is an infinite number of rational and irrational numbers.
- b. Adapt the proof that $\sqrt{2}$ is an irrational number to argue that $\sqrt{7}$, $\sqrt{14}$ and $\sqrt{2} + \sqrt{7}$ are irrational numbers
- c. Prove that if p is a prime number, then \sqrt{p} is an irrational number

2. Mathematical Chairs (This mathematical version of “musical chairs” is 2000 years old.) A math class with “ n ” students arranges n chairs in a circle and numbers the chairs clockwise from 1 to n . Each student sits in one of the chairs. An *elimination number* “ d ” is chosen. Starting with the student in Chair 1, students “count off” 1, 2, 3, ..., d . The student in Chair d is then removed from the circle (i.e., eliminated from the game). The next student (at this point it would be the student in Chair $d+1$, starts the count over starting with 1. Again, the students count off, in order, 1, 2, 3, ..., d , and the student who counts “ d ” is eliminated from the game. When the count gets to the student in Chair n , the next person to count is the student in Chair 1. (If, for example, a class has 10 students and the elimination number is 3, the first students to be eliminated would be those in Chairs 3, 6, and 9 followed by the students in Chair 2 and Chair 7.) The “winner” of Mathematical Chairs is the last student sitting in the circle when all the other students have been eliminated.

- a. Play the game using 2 as the elimination number in your math class. Where should a student sit in order to win the game? (Your solution should indicate n , the number of students in your class.
- b. Build a table that gives the “winning chair” for different values of n and elimination number 2.
- c. Build additional tables that give the winning seat for different values of n and an elimination number of 3 and then 4.
- d. Solve the general Mathematical Chairs game for an elimination number of 2. (Can you solve a more general Mathematical Chairs game for n students and elimination number d ?)

3. What new knowledge of prime numbers do you have as a result of Math in the Middle? Discuss how you have used this knowledge to enrich a math course you teach, or, if you haven't made use of this knowledge, develop a plan for how you will use your knowledge of prime numbers and discuss the impact that you hope it will have in your class.

Part IB. For this part of the exam, you should research the topic below and write an expository paper (including proofs or examples where appropriate). It is anticipated that your paper will be approximately 8 to 10 pages.

Conic Sections

The ellipse, the parabola, and the hyperbola are known collectively as the *conic sections*. Your paper should discuss the mathematical properties of the conic sections, using the following suggestions as a guideline.

1. You could begin with a geometric description of each curve. The parabola is defined in terms of a point called the *focus* and a line called the *directrix*. The ellipse can be defined using a focus and directrix definition or in terms of two focal points (two *foci*). Similarly, the hyperbola can be defined using a focus and directrix or in terms of two foci. What are these definitions?
2. Describe physical models for each curve. The ellipse can be traced out with a loop of string around two fixed pins. The parabola can be approximated by a paper-folding process, or by sliding a short straight-edge inside a shoe box lid. What about the hyperbola?
3. Give simple algebraic expressions whose graphs are examples of the conic sections. For example, the graph of $y = x^2$, is a parabola. Give similar simple expressions for the ellipse and the hyperbola.
4. Generalize the expressions in Part 3 above. Connect the definitions in Part 1 with possible graphs of a 2nd degree polynomial equation in two variables. This will take some thought. (You should probably work out the algebraic details with paper and pencil before trying to format them in your word processor.) This section could be the longest part of your paper. You may need to introduce some new definitions, such as *vertex* of a parabola, or *eccentricity* of an ellipse or hyperbola.
5. Explain with a diagram or model why these curves are called “conic sections.”
6. Describe some practical applications of the conic sections. Where do these curves occur in the natural world? What are some industrial or scientific applications for these curves?
7. Include a reference section using a standard bibliographic format.

Part II. For the oral part of your exam, you should prepare a presentation based on your paper in Part IB. Your talk should also discuss your journey through Math in the Middle, emphasizing one aspect of your Math in the Middle experience:

- mathematics
- pedagogical preparation
- action research
- leadership development

The presentation should be approximately 20 minutes in length. Following your presentation, expect the committee to be interested in your work and your experiences in Math in the Middle. Thus, you should expect them to ask a few questions that raise interesting points about what you have presented. For example, the committee may ask questions designed to probe your depth of understanding of the topic about which you have written and the committee may ask questions related to the answers you provided for Part IA. In the past, most MAT candidates use a computer presentation program such as Power Point, although a prepared set of overhead transparencies or a handout is acceptable. Don't overload your slides with too many words and pictures.