

Trains

The set of problems below revolve around “trains” of rods. You can use rods of integer sizes to build “trains” that all have a common length. For instance, a “train of length 5” is a row of rods whose combined length is 5. Here are some examples:

1	2	2
2	1	2
1	3	1
1	4	
5		

Notice that the 1-2-2 train and the 2-1-2 train contain the same rods but are listed separately. If you use identical rods in a different order, it is considered a separate train.

Important Stuff

1. How many trains of length 5 are there?
2. Complete the following table that gives the number of trains of length n .

Length n	1	2	3	4	5	6
# of Trains					16	

3. Find a formula for the number of trains of length n . Come up with a convincing reason that your rule is correct.
4. Create an algorithm that will generate all trains of length n .
5. How many trains of length 6 are there that use exactly one rod? two rods? three rods?
6. (a) Make a table for $n = 2$ to 8 for the number of trains of length n that use *exactly* two rods.
(b) Repeat for *exactly* three rods.
7. Complete this table, with the train length on the vertical and the number of rods on the horizontal:

		# Rods Used						
		1	2	3	4	5	6	7
1		1						
2		1	1					
3		1		1				
4		1			1			
5		1				1		
6		1	5	10			1	
7		1						1

8. Physically make all the trains of length 6 that use exactly two rods, and (separately, without destroying the two-rod trains) all the trains of length 6 that use exactly three rods.
9. Can you think of a way to use the trains you built in Problem 8 to make all the trains of length 7 that use exactly three rods?

Neat Stuff

10. Sarah sets you up with a whole lot of red rods (length 2).
 - (a) Find all the ways to make a 4-by-2 rectangle using red rods. Note that a 4-by-2 rectangle has length 4 and width 2 only; no 2-by-4 rectangles qualify!
 - (b) Complete this table that gives the number of ways to use red rods to make n -by-2 rectangles for increasing values of n :

Length n	1	2	3	4	5	6	7	8
# Ways								

11. How many trains of length 8 are there that use *only* cars of length 1 and 2?
12. How many trains of length n are there that use *only* cars of length 1 and 2? Find a general rule, and explain why your rule works.
13. Using *only* rods of lengths 2 and 3, how many trains of length 11 can be made?
14. Make a table of how many trains of length n can be made using *only* rods of length 2 and 3, for n from 1 to 11. Is there a rule you could use to continue the table?

15. You've got an unlimited supply of rods 2, 3, and 5 (but no other types of rods). How many different trains of length 12 can you make with all these rods?
16. Suppose you have an unlimited supply of only rod lengths 3, 4, and 8. Find the total number of ways to make a train of length 14, using any method you like.

Tough Stuff

17. How many trains of length 10 can you make with *no* rods of length 1?
18. Suppose you want to make all the trains of length 3, but not all at the same time. You want to make them one at a time. How many of each car do you need? Well, here are the trains:

1-1-1, 1-2, 2-1, 3.

You need three 1-cars, one 2-car (because any given train only uses one of them), and one 3-car.

How many cars (and which ones) do you need on your desk to make all the trains of length 4, doing it one train at a time? Now suppose you want to make all the trains of length 5, one at a time. What do you need to *add* to the pile on your desk so you can do it? Then how many cars do you need to add to the pile in order to make all the trains of length 6? Generalize to length n : How many new cars do you need to add to a pile that lets you make all trains of length $n - 1$ in order to get a pile that lets you make all trains of length n ?

19. If you made all the trains of length 5, how many cars of length 1 were used? length 2, 3, 4, 5? See if you can find a general rule for the number of cars of length k you'd need to make all the trains of length n .
20. What is the *average* (mean) length of car used when you make all the trains of length 5? Is there a general rule at work here? Can you justify it?