The Simplex Lock

The Simplex company makes a combination lock that is used in many public buildings. It comes in several versions. Here is one:

These 5-button devices are purely mechanical (no electronics). You can set the combination using the following rules:

1. A combination is a sequence of zero or more pushes, each push involving at least one button.
2. Each button may be used at most once (once you press it, it stays in).
3. Each push may include any of the buttons that haven't been pushed yet, up to and including all remaining buttons.
4. The combination does *not* need to include all buttons.
5. When two or more buttons are pushed at the same time, order doesn't matter.

Here's an artist's rendition of a Simplex Lock.

And here are some possible combinations:

- $\{\{1,3\},\{4\}\}$
- $\{\{1,2\},\{3\}\}$
- $\{\{2\},\{1\},\{3\}\}$
- $\{\{1,2,4\},\{3,5\}\}$
- $\{\{1,2,3,4,5\}\}$
- $\{\{1,2\},\{4\},\{3,5\}\}$
- $\{\{3\},\{1,2\}\}$
- $\{\ \}$
- $\{2\}$

The company advertises thousands of combinations, and (as we say to our students) the question is, "Is the company telling the truth?"

Problem. How many combinations are there on a 5-button Simplex Lock?